LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2011

MT 5407 - FORMAL LANGUAGES AND AUTOMATA

Date : 12-11-2011 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART – A

Answer ALL questions (10 x 2 = 20)

1. Define a finite automaton.
2. Construct the state diagram for the automaton $M = (\{q_0, q_1, q_2\}, \{a,b\}, \delta, q_0, \{q_2\})$ where δ is given by

$$\delta(q_0, a) = q_1 \qquad \delta(q_1, a) = q_1$$
$$\delta(q_2, a) = q_1 \qquad \delta(q_0, b) = q_2$$
$$\delta(q_1, b) = q_2 \qquad \delta(q_2, b) = q_0$$

3. Define a non deterministic finite automaton.
4. Prove that any finite subset is regular.
5. Define context-sensitive language.
6. Write a grammar for the language $L = \{a^n b^n / n \geq 1\}$.
7. Define concatenation of two languages.
8. Define an $\varepsilon -$ free homomorphism.
9. State the Chomsky Normal form for the regular expressions.
10. Define ambiguously derivable.

PART – B

Answer any FIVE questions (5 x 8 = 40)

11. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton. Let R be a relation in Q defined by $q_1 R q_2$ if $\delta(q_1, a) = \delta(q_2, a)$ for all a in Σ. Show that R is an equivalence relation.
12. Construct a finite automaton that accepts exactly those input strings of 0's and 1's that end in 11.
13. $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\})$ is a finite automaton and δ is given by

$Q \setminus \Sigma$	0	1
q_0	q_1	q_1
q_1	q_2	q_3
q_2	q_3	q_1
q_3	q_0	q_2

Find (i) $\hat{\delta}(q_0, 011010)$

(ii) $\hat{\delta}(q_0, 1110011)$

14. Prove that union of two regular sets is also regular.
15. Let $G = (N, T, P, S)$, $N = \{S, B\}$, $T = \{a, b, c\}$. P consists of the following productions:

   1. $S \rightarrow aSBc$ 3. $cB \rightarrow Bc$
   2. $S \rightarrow abc$ 4. $bB \rightarrow bb$

   Then show that $L(G) = \{a^n b^n c^n / n \geq 1\}$ is a CSL.
16. Write a CNF grammar for the language $L = \{wcw^R / w \in (a,b)^*\}$ and give two examples.
17. Prove that if L is a CFL generated by $G = (N, T, P, S)$, where P consists of rules of the form $A \rightarrow \alpha$, $A \in N$, $\alpha \in (N \cup T)^*$, then L can be generated by a CFG in which every rule is either of the form $A \rightarrow \alpha$, $A \in N$, $\alpha \in (N \cup T)^+$, or $S \rightarrow \varepsilon$, Further S does not appear on the right side of any rule.
18. Let $G = (N, T, P, S)$, where $N = \{S, A\}$, $T = \{a, b\}$ and P consists of the rules

   1. $S \rightarrow aAb$ 2. $S \rightarrow abSb$ 3. $S \rightarrow a$
   4. $A \rightarrow bS$ 5. $A \rightarrow aAAb$

   Find the leftmost and rightmost derivations for the string $abab$.

PART – C

Answer any TWO question (2 x 20 = 40)

19. a) Let $M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_1\})$ where δ is given by

$\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_2$

$\delta(q_1, a) = q_3$ $\delta(q_1, b) = q_0$

$\delta(q_2, a) = q_2$ $\delta(q_2, b) = q_2$

$\delta(q_3, a) = q_2$ $\delta(q_3, b) = q_2$

(a) Construct the state table for the given automaton M.

(b) Draw the state diagram for the given automaton M.

(c) Which of the following strings are accepted by M?

(i) ababa (ii) aabba (iii) aaaab (iv) bbbaa

b) Construct a finite automaton M accepting $\{ab, ba\}$. (15+5)

20. State and prove the Pumping Lemma. (20)

21. Let $G = (\{S, Z, A, B\}, \{a, b\}, P, S)$ where P consists of the following productions:

1. $S \to aSA$	4. $Z \to bB$	7. $bB \to bb$
2. $S \to aZA$	5. $BA \to AB$	8. $bA \to ba$
3. $Z \to bZB$	6. $AB \to Ab$	9. $aA \to aa$

Then show that $L(G) = \{a^n b^m a^n b^m \,/\, n, m \geq 1\}$. (20)

22. a). Prove that the family of CFL is closed under substitution.

b). Let $G = (N, T, P, S)$, be any context-free grammar generating a non- empty language. Show that there exists an equivalent grammar G_1 such that for each non –terminal A of G_1, there is a derivation $S \overset{*}{\Rightarrow} \alpha_1 A \alpha_2$, $\alpha_1, \alpha_2 \in (NUT)^*$. (10+10)

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